Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel Level 3 GCE**

| Centre Number | Candidate Number |
|---|---|
| | |

**Time** 2 hours

**Paper reference** **8MA0/01**

# Mathematics

## Advanced Subsidiary
## PAPER 1: Pure Mathematics

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

*Turn over*

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Pearson

**1.** **In this question you should show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

**(3)**

Step 1: Get 0 on one side

$x^2 - x - 20 > 0$

Step 2: Solve as an equality

$x^2 - x - 20 = 0$

$(x - 5)(x + 4) = 0$

$x = 5,\ x = -4$

Step 3: Solve the inequality

Way 1: Number line with sign change test


try $x = -5$ or any number less than $-4$. plug into $(x-5)(x+4)$

$(-9)(-1)$

$= 9$

$+$

try $x = 0$: or any number between $-4$ and $5$. plug into $(x-5)(x+4)$

$(-5)(4)$

$= -20$

$-$

try $x = 6$ or any number greater than $5$. plug into $(x-5)(x+4)$

$(1)(10)$

$= 10$

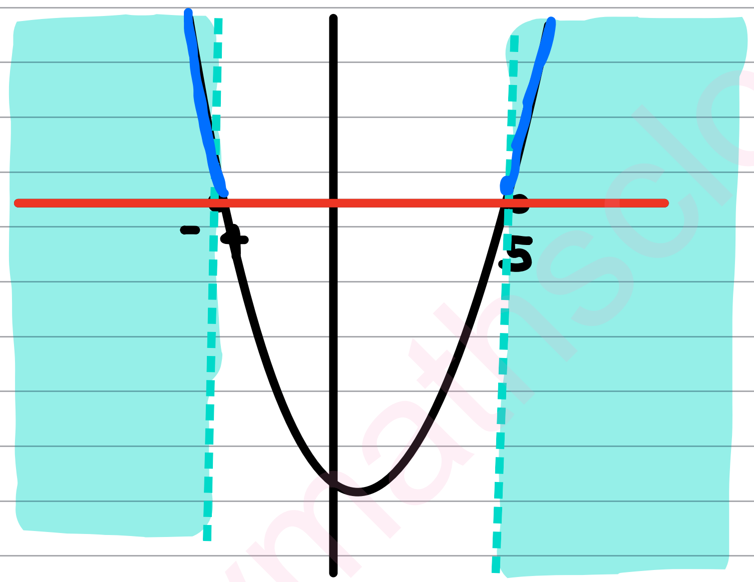
$+$

question wanted $>0$ which means positive so we want $x$ region where have +

$x < -4$ or $x > 5$

$\{x : x < -4 \cup x > 5\}$

Way 2: Graph $(x-5)(x+4)$




We are solving $(x-5)(x+4) > 0$

$>0$ means want where above $x$ axis

$x$ values

$x < -4$ or $x > 5$

$\{x : x < -4 \cup x > 5\}$

**(Total for Question 1 is 3 marks)**

2. **In this question you should show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express $y$ in terms of $x$, writing your answer in simplest form.

**(3)**

Way 1: Make all the bases the same

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

$$\frac{(3^2)^{x-1}}{3^{y+2}} = 3^4$$

Use indices rule $(x^a)^b = x^{ab}$

$$\frac{3^{2x-2}}{3^{y+2}} = 3^4$$

Use indices rule $\frac{x^a}{x^b} = x^{a-b}$

$$3^{2x-2-(y+2)} = 3^4$$

$$3^{2x-2-y-2} = 3^4$$

$$3^{2x-y-4} = 3^4$$

We have the same base on each side, so the powers must be equal

$$2x-y-4 = 4$$

$$y = 2x-8$$

**Question 2 continued**

Way 2: take logs of both sides

$$\log \frac{9^{x-1}}{3^{y+2}} = \log 81$$

Use log rule $\log \frac{a}{b} = \log a - \log b$

$$\log 9^{x-1} - \log 3^{y+2} = \log 81$$

Use log rule $\log a b^{c} = c \log ab$

$$(x-1)\log 9 - (y+2)\log 3 = \log 81$$

$$(\log 9)x - \log 9 - (\log 3)y - 2\log 3 = \log 81$$

choose base 3 for all logs

$$(\log_3 9)x - \log_3 9 - (\log_3 3)y - 2\log_3 3 = \log_3 81$$

$$2x - 2 - y - 2 = 4$$

$$y = 2x - 8$$

**(Total for Question 2 is 3 marks)**



P 6 6 5 8 5 A 0 5 4 4

3. Find

$$\int \frac{3x^4 - 4}{2x^3}\,dx$$

writing your answer in simplest form.

**(4)**

Way 1: $\int \frac{3x^4-4}{2x^3}\,dx$

Get into the best form to integrate (single terms - no fractions)

$= \int \left(\frac{3x^4}{2x^3} - \frac{4}{2x^3}\right) dx$

use indices rule $\frac{1}{x^n} = x^{-n}$

$= \int \left(\frac{3x}{2} - \frac{2}{x^3}\right) dx$

$= \int \frac{3}{2}x - 2x^{-3}) \, dx$

Add 1 to the power and divide by it

$= \frac{3}{2} \times \frac{x^2}{2} - 2\frac{x^{-2}}{-2} + c$

$= \frac{3}{4}x^2 + x^{-2} + c$

$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$

Question 3 continued

Way 2: $\int \frac{3x^4-4}{2x^3}\,dx$

$= \int \frac{(3x^4-4)}{2} x^{-3}\,dx$

Write as $\frac{3x^4-4}{2} \times \frac{x^{-3}}{1}$ and use fraction rules to multiply

Use indices rules to multiply $(3x^4-4)x^{-3}$ i.e add the powers

$= \int \frac{3x - 4x^{-3}}{2}\,dx$

$= \int \left(\frac{3}{2}x - \frac{4}{2}x^{-3}\right)dx$

Integrate - Add 1 to the power and divide by it

$= \frac{3}{2} \times \frac{x^2}{2} - \frac{4}{2} \times \frac{x^{-2}}{-2} + c$

$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$

**(Total for Question 3 is 4 marks)**

**4.** [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point $O$.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point $O$.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through $O$, **(2)**

(b) calculate the speed of the stone. **(3)**

a) Everything is relative to a fixed point O, so it makes sense to use the origin

$\vec{AO} = O - A = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -24 \\ -10 \end{pmatrix} = \begin{pmatrix} 24 \\ 10 \end{pmatrix}$

We always do 2nd letter - 1st letter

$\vec{BO} = O - B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \begin{pmatrix} -12 \\ -5 \end{pmatrix}$

Now compare $\vec{AO}$ and $\vec{BO}$

$\vec{AO} = -2\vec{BO}$ hence parallel

The stone is travelling in a straight line hence must pass through O

Question 4 continued

b) $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 5 \end{pmatrix} - \begin{pmatrix} -24 \\ -10 \end{pmatrix} = \begin{pmatrix} 12+24 \\ 5+10 \end{pmatrix} = \begin{pmatrix} 36 \\ 15 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{36^2 + 15^2} = 39$ (distance)

$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{39}{4}$

$= 9.75 \text{ ms}^{-1}$

**(Total for Question 4 is 5 marks)**

**5.**

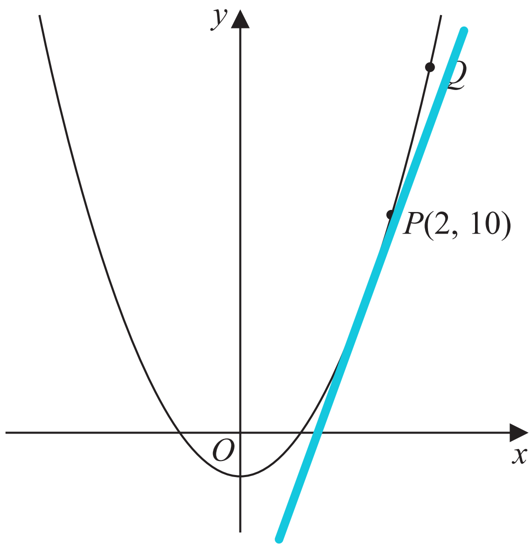



**Figure 1**

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at $P$.

**(2)**

The point $Q$ with $x$ coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line $PQ$, giving your answer in terms of $h$ in simplest form.

**(3)**

(c) Explain briefly the relationship between part (b) and the answer to part (a).

**(1)**

a) Tangent equation: $y - y_1 = m(x - x_1)$

or can use $y = mx + c$

Step 1: find slope m by differentiating and plugging in the point

$\frac{dy}{dx} = 6x$

given the point $(2, 10)$

when $x = 2$: $\frac{dy}{dx} = 6(2) = 12$

b) $P(2, 10)$ $\quad Q(2+h, y)$

to find $y$:
plug $x = 2+h$ into $y = 3x^2 - 2$

$$y = 3(2+h)^2 - 2$$
$$= 3(4+4h+h^2) - 2$$
$$= 12 + 12h + 3h^2 - 2$$
$$= 3h^2 + 12h + 10$$

$$\text{Gradient } PQ = \frac{3h^2+12h+10-10}{2+h-2} = \frac{3h^2+12h}{h}$$
$$= 3h + 12$$

c) What we have found in b) is the differentiation by first principles formula without the limit part

let $h \to 0$

$3h + 12 \to 3(0) + 12 = 12$

The gradient of the chord tends to the gradient of the tangent to the curve i.e the gradient of the curve at the point $x = 2$ is 12.

**(Total for Question 5 is 6 marks)**

**Question 5 continued**

**(Total for Question 5 is 6 marks)**

6. **In this question you should show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

**(3)**

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

**(3)**

a)

$3x^3 - 17x^2 - 6x = 0$

Factorise

$x(3x^2 - 17x - 6) = 0$

$x(3x+1)(x-6) = 0$

Set each bracket equal to zero

$x = 0,\ 3x+1 = 0,\ x-6 = 0$

$x = 0,\ x = -\frac{1}{3},\ x = 6$

b) This is closely linked to part a)

$3x^3 - 17x^2 - 6x = 0$

$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$

let $x = (y-2)^2$

So our answers from a) become

**Question 6 continued**

$(y-2)^2 = 0, \quad \underbrace{(y-2)^2 = -\frac{1}{3}}_{\text{no soln since can't root a neg number}}, \quad (y-2)^2 = 6$

$y - 2 = 0$

$y = 2$

$y - 2 = \pm\sqrt{6}$

$y = 2 \pm \sqrt{6}$

$y = 2, \; 2 \pm \sqrt{6}$

**(Total for Question 6 is 6 marks)**

**7.** A parallelogram $PQRS$ has area $50\,\text{cm}^2$

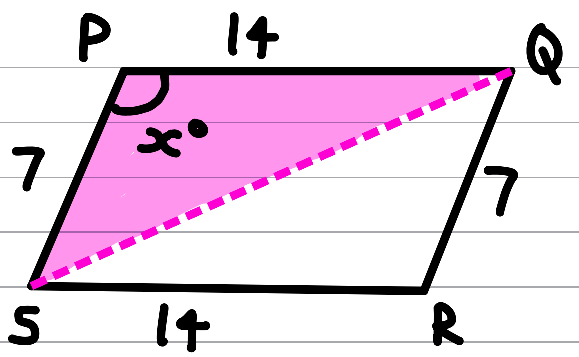
Given

- $PQ$ has length 14 cm
- $QR$ has length 7 cm
- angle $SPQ$ is obtuse

find

(a) the size of angle $SPQ$, in degrees, to 2 decimal places, **(3)**

(b) the length of the diagonal $SQ$, in cm, to one decimal place. **(2)**




a) Area formula $= 2\left[\frac{1}{2}ab\sin x\right] = ab\sin x$

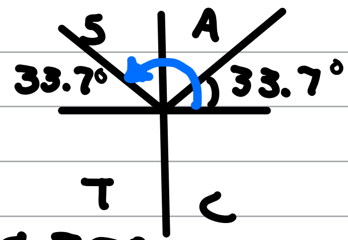
$$\text{Area} = (7)(14)\sin x = 50$$

$$98\sin x = 50$$

$$\sin x = \frac{50}{98}$$

$$x = \sin^{-1}\left(\frac{50}{98}\right)$$

$$= 30.677°$$




Obtuse angle $= 180 - 30.677°$

$$= 149.32°$$

**Question 7 continued**

b) use cosine rule on triangle PSQ

$x^2 = 7^2 + 14^2 - 2(7)(14)\cos 149.32$

$x^2 = 413.565964$

$x = 20.3$ cm

**(Total for Question 7 is 5 marks)**





P 6 6 5 8 5 A 0 1 5 4 4

**8.** $\quad g(x) = (2 + ax)^8 \qquad$ where $a$ is a constant

Given that one of the terms in the binomial expansion of g($x$) is $3402x^5$

(a) find the value of $a$.

**(4)**

Using this value of $a$,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

**(3)**

a) $g(x) = (2+ax)^8$

Need to find the $x^5$ term

We know that each term in the expansion looks like:

$^8C_?\,(2)^?\,(ax)^?$

these powers add to make 8

these match

Last bracket must have a power of 5 since want $x^5$

Let's use the pattern to fill the rest in.

$^8C_5\,(2)^3\,(ax)^5$

$= 56(8)a^5x^5$

$= \underbrace{448a}\,{}^5x^5$

$448a^5 = 3402$





P 6 6 5 8 5 A 0 1 6 4 4

$a^5 = \dfrac{3402}{448}$

$a^5 = \dfrac{243}{32}$

$a = \dfrac{3}{2}$

b) constant term is the same as saying $x^0$ term

$\left(1+\dfrac{1}{x^4}\right)\left(2+\dfrac{3}{2}x\right)^8$

needs $x^4$ term from expansion

Needs $x^0$ term from expansion

$x^0$ term from : ${}^8C_0(2)^8\left(\frac{3}{2}x\right)^0$

$= (1)(256)(1)$

$= 256$

$x^4$ term from : ${}^8C_4(2)^4\left(\frac{3}{2}x\right)^4$

$= 70(16)\left(\dfrac{81}{16}\right)x^4$

$= 5670x^4$

So we have $\left(1+\dfrac{1}{x^4}\right)\left(256+5670x^4+\ldots\right)$

**(Total for Question 8 is 7 marks)**

Question 8 continued

We only worry about multiplying the orange and green terms separately since the other combinations won't give the constant term

$= 256 + 5670$

$= 5926$

(Total for Question 8 is 7 marks)

**9.** Find the value of the constant $k$, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

**(4)**

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx$$

$$= \int_k^9 6x^{-\frac{1}{2}}\,dx$$

$$= \left[\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}\right]_k^9$$

$$= \left[12\sqrt{x}\right]_k^9$$

$$= 12\sqrt{9} - 12\sqrt{k}$$

$$= 12(3) - 12\sqrt{k}$$

$$= 36 - 12\sqrt{k}$$

$$36 - 12\sqrt{k} = 20$$

$$12\sqrt{k} = 16$$

$$\sqrt{k} = \frac{16}{12}$$

$$\sqrt{k} = \frac{4}{3}$$

$$k = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

**10.** A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers. **(4)**

(b) Use a counterexample to show that the statement is not always true. **(1)**

a)

consider n odd

This means n has the form $2k+1$

$n^3 - n = (2k+1)^3 - (2k+1)$

$= (2k+1)(2k+1)(2k+1) - (2k+1)$

$= (4k^2+4k+1)(2k+1) - (2k+1)$

$= 8k^3 + 4k^2 + 8k^2 + 4k + 2k + 1 - 2k - 1$

$= 8k^3 + 12k^2 + 4k$

$= 4(2k^3 + 3k^2 + 1)$

= multiple of 4 since $4 \times$ something

b) $n^3 - n$

let $n = 2$

$2^3 - 2$

$= 8 - 2$

$= 6$

this is not a multiple of 4





P 6 6 5 8 5 A 0 2 0 4 4

**11.** The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, $A$ km$^2$, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where $c$ is a constant and $t$ is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

**(1)**

On 1st January 2019 an area of 60 km$^2$ of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of $c$ to 3 significant figures.

**(4)**

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km$^2$ of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

**(1)**

$A = 80 - 45e^{ct}$

a) let $t = 0$

$A = 80 - 45e^{c(0)}$

$A = 80 - 45e^{0}$

$A = 80 - 45(1)$

$A = 80 - 45$

$A = 35$ km$^2$

b) when $t = 14$, $A = 60$

$60 = 80 - 45e^{14c}$

$45e^{14c} = 20$



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**Question 11 continued**

$e^{14c} = \frac{20}{45}$

ln both sides

$\ln e^{14c} = \ln \frac{20}{45}$

$14c = \ln \frac{20}{45}$

$c = \frac{\ln \frac{20}{45}}{14} = -0.0579$

so $A = 80 - 45e^{-0.0579t}$

c) let $A = 100$

$100 = 80 - 45e^{-0.0579t}$

$45e^{-0.0579t} = -20$

$e^{-0.0579t} = -\frac{4}{9}$

We can solve this since we can't take the natural log of a negative number, hence no solution $\therefore$ model not appropriate

**(Total for Question 11 is 6 marks)**

12. **In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

**(5)**

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

**(2)**

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$

(b) Find, to the nearest degree, the value of $\alpha_4$

**(2)**

i) $5\cos^2\theta = 6\sin\theta$

$5(1-\sin^2\theta) = 6\sin\theta$

$5 - 5\sin^2\theta = 6\sin\theta$

$5\sin^2\theta + 6\sin\theta - 5 = 0$

let $\sin\theta = y$

$5y^2 + 6y - 5 = 0$

won't factorise so need to use quadratic formula

$$y = \frac{-3 \pm \sqrt{34}}{5}$$

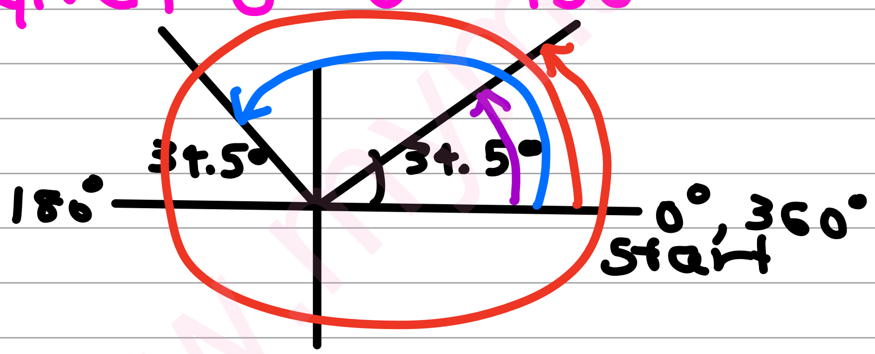
$$\Rightarrow \sin\theta = \frac{-3+\sqrt{34}}{5}, \quad \sin\theta = \frac{-3-\sqrt{34}}{5}$$

$= 0.56619$ , $= -1.76619$ (no solution)

$\theta = \sin^{-1}(0.56619)$

$= 34.5°$

given $0 < \theta \leq 450°$




$\theta = 34.5, 145.5°, 394.5°$

ii)

a) $3\sin x - 5\sin x\cos x = 0$

$\sin x(3 - 5\cos x) = 0$

$\sin x = 0, \cos x = \frac{3}{5}$

error 1: missing



Turn over

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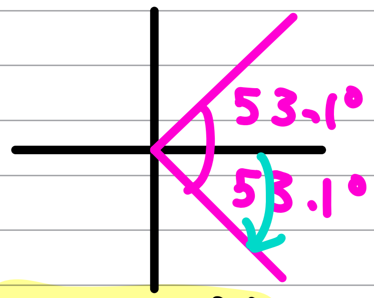
Question 12 continued $\cos^{-1}\left(\frac{3}{5}\right) = 53.1°$




Limits: $-90° < x < 90°$

error 2:

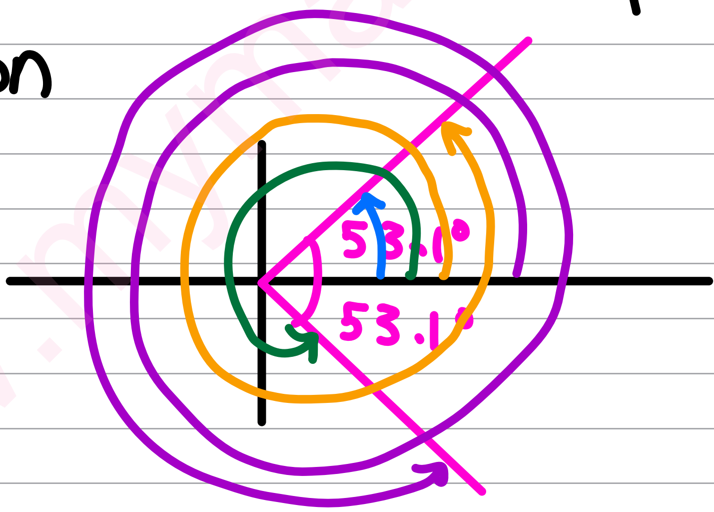
Missing the solution $-53.1°$

b) $\cos x = \frac{3}{5} \Rightarrow \cos(5\alpha + 40°) = \frac{3}{5}$

angle changes

Not given limits now

We want the fourth largest positive solution




$5\alpha + 40 = 720 - 53.1$

$5\alpha + 40 = 566.9$

$5\alpha = 626.9$

$\alpha = 125°$

**13.**

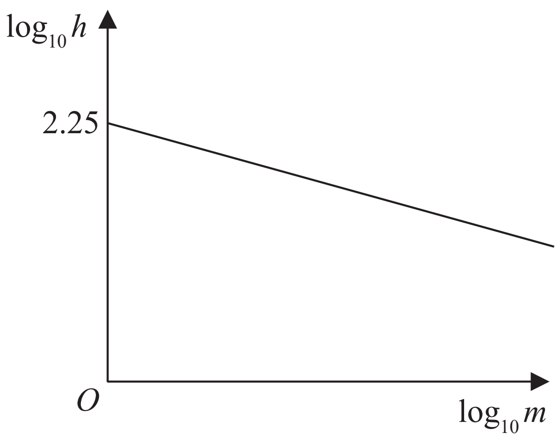



**Figure 2**

The resting heart rate, $h$, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where $p$ and $q$ are constants and $m$ is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of $-0.235$

(a) Find, to 3 significant figures, the value of $p$ and the value of $q$.

**(3)**

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

**(3)**

(c) With reference to the model, interpret the value of the constant $p$.

**(1)**

a) $h = pm^q$

lets log both sides to turn this into a linear $y = mx + c$ model

$\log h = \log pm^q$

use log rule $\log ab = \log a + \log b$

$\log h = \log p + \log m^q$





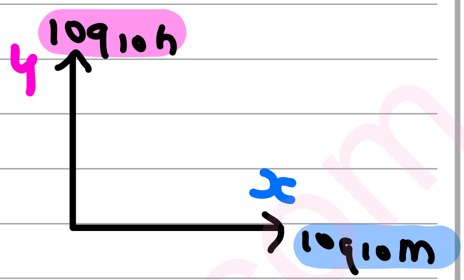
P 6 6 5 8 5 A 0 3 0 4 4

**Question 13 continued**

Use log rule $\log m^q = q\log m$

$\log h = \log p + q\log m$

No base means base 10 by default




$\log_{10} h = \log_{10} p + q\log_{10} m$

$\log_{10} h = q\log_{10} m + \log_{10} p$ ①

Looks like a $y = mx + c$ model

given gradient $= -0.235$

given y intercept 2.25

so $y = -0.235x + 2.25$ ②

Let's compare ① and ②

$q = -0.235$ and $\log_{10} p = 2.25$

Use log rule $\log_a b = c \Leftrightarrow a^c = b$

$10^{2.25} = p$

$p = 178$

$p = 178,\ q = -0.235$

**Question 13 continued**

b) $h = pm^q$

$h = 178m^{-0.235}$

let $m = 5$

$h = 178(5)^{-0.235} = 121.944 = 122$ 3 s.f.

$122 \approx 119$ (accurate to 2 s.f)

$\therefore$ model is suitable

c) p is the resting heart rate in bpm of a mammal with a mass of 1 kg

**14.** A curve $C$ has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x+b)^2 + c$$

where $a$, $b$ and $c$ are constants to be found.

**(3)**

The curve $C$ has a maximum turning point at $M$.

(b) Find the coordinates of $M$.

**(2)**

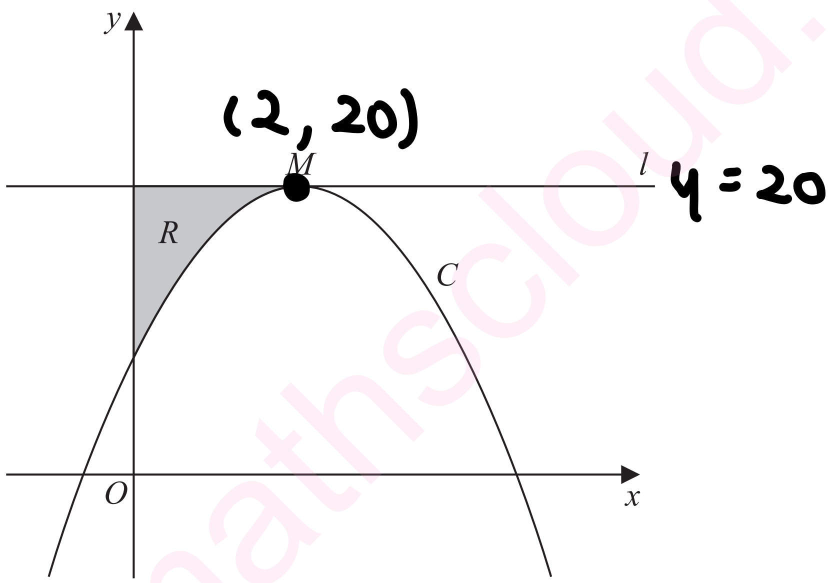



**Figure 3**

Figure 3 shows a sketch of the curve $C$.

The line $l$ passes through $M$ and is parallel to the $x$-axis.

The region $R$, shown shaded in Figure 3, is bounded by $C$, $l$ and the $y$-axis.

(c) Using algebraic integration, find the area of $R$.

**(5)**

a) we need to complete the square

$-3x^2+12x+8$

$-3(x^2-4x)+8$

$-3[(x-2)^2-4]+8$



P 6 6 5 8 5 A 0 3 4 4 4

Question 14 continued

$-3(x-2)^2+12+8$

$-3(x-2)^2+20$

b) $(2,20)$

(2)

c)

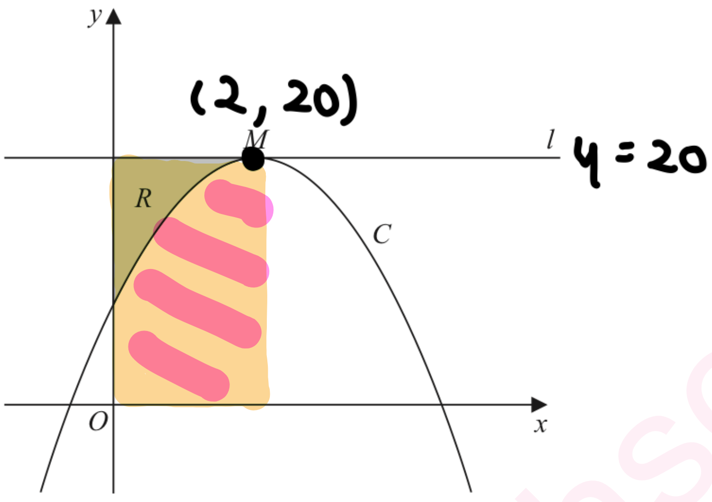



Figure 3

$\int_0^2 20\,dx - \int_0^2 (-3x^2+12x+8)\,dx$

$= [20x]_0^2 - \left[-\frac{3x^3}{3}+\frac{12x^2}{2}+8x\right]_0^2$

$= [20x]_0^2 - \left[-x^3+6x^2+8x\right]_0^2$

$=(40-0) - [(-2^3+6(2)^2+8(2)) - 0)]$

$= 40 + 8 - 24 - 16$

$= 8$

**Question 14 continued**

Note: Instead of integrating to find R we could have found the area of the rectangle
$= 2(20) = 40$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

**Question 14 continued**

**(Total for Question 14 is 10 marks)**

**15.**

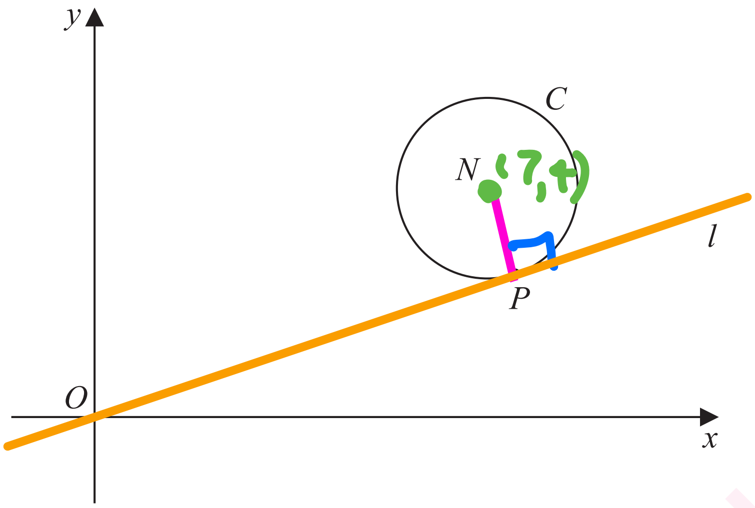



**Figure 4**

Figure 4 shows a sketch of a circle $C$ with centre $N(7, 4)$

The line $l$ with equation $y = \frac{1}{3}x$ is a tangent to $C$ at the point $P$.

Find

(a) the equation of line $PN$ in the form $y = mx + c$, where $m$ and $c$ are constants,

**(2)**

(b) an equation for $C$.

**(4)**

The line with equation $y = \frac{1}{3}x + k$, where $k$ is a non-zero constant, is also a tangent to $C$.

(c) Find the value of $k$.

**(3)**

a) We need the slope and a point

slope of tangent $= \frac{1}{3}$

PN is perpendicular to tangent (since tangent meets a radius at 90°)

$\Rightarrow$ slope PN $= -3$

Let's use point $(7, 4)$

Question 15 continued

Plug the gradient and point into

$y - y_1 = m(x - x_1)$

$y - 4 = -3(x - 7)$

$y - 4 = -3x + 21$

$y = -3x + 25$

b) Need to find point P in order to find the radius length

P is where $y = \frac{1}{3}x$ meets $y = -3x + 25$

We solve simultaneously

$\frac{1}{3}x = -3x + 25$

$x = -9x + 75$

$10x = 75$

$x = 7.5$

plug into either equation

$y = \frac{1}{3}(7.5) = 2.5$

We can now find length PN: P $(7.5, 2.5)$

N$(7, 4)$

length $PN = \sqrt{(7.5-7)^2+(2.5-4)^2}$

$= \sqrt{2.5}$

centre $(7,4)$

Equation of circle: $(x-7)^2+(y-4)^2=2.5$

c) $y=\frac{1}{3}x+k$, $(x-7)^2+(y-4)^2=2.5$

Way 1:

• tangent $\Rightarrow$ intersects

Solve simultaneously

$(x-7)^2+\left(\frac{1}{3}x+k-4\right)^2=2.5$

• tangent to circle $\Rightarrow$ intersects once

Therefore we can use the discriminant $=0$ (1 real root)

We need to re-arrange before we can use the discriminant. Want the form $ax^2+bx+c=0$

$(x-7)^2+\left(\frac{1}{3}x+k-4\right)^2=2.5$

$\Rightarrow x^2-14x+49+\frac{1}{9}x^2+\frac{1}{3}xk-\frac{4}{3}x$

$+\frac{1}{3}kx+k^2-4k-\frac{4}{3}x-4k+16=2.5$

**Question 15 continued**

Group the $x^2$, $x$ and constant terms

$\Rightarrow \frac{10}{9}x^2 + \left(-14 + \frac{1}{3}k - \frac{4}{3} + \frac{1}{3}k - \frac{4}{3}\right)x$

$+ \left(49 + k^2 - 4k - 4k + 16 - 2.5\right) = 0$

$\Rightarrow \frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + (k^2 - 8k + 62.5) = 0$

$a = \frac{10}{9}$ $\quad b = \frac{2}{3}k - \frac{50}{3}$ $\quad c = k^2 - 8k + 62.5$

$b^2 - 4ac = 0$

$\left(\frac{2}{3}k - \frac{50}{3}\right)^2 - 4\left(\frac{10}{9}\right)(k^2 - 8k + 62.5) = 0$

$\frac{4}{9}k^2 - \frac{200}{9}k + \frac{2500}{9} - \frac{40}{9}k^2 + \frac{320}{9}k - \frac{2500}{9} = 0$

$4k^2 - 200k + 2500 - 40k^2 + 320k - 2500 = 0$

$36k^2 - 120k = 0$

$4k(9k - 30) = 0$

$k = 0, \; k = \frac{10}{3}$

$\downarrow$

picks up the $y = 2x$ we already know $\qquad k = \frac{10}{3}$

**(Total for Question 15 is 9 marks)**

# c)In more detail

## Way 1: Use discriminant

Tangent means meets once hence discriminant $= 0$. First we need to build the equation (representing intersection) to use the discriminant on

$$y = \frac{1}{3}x + k$$

$$(x-7)^2 + (y-4)^2 = 2.5$$

We solve simultaneously

We can sub $y = \frac{1}{3}x + k$ into $(x-7)^2 + (y-4)^2 = 2.5$

$$(x-7)^2 + \left(\frac{1}{3}x + k - 4\right)^2 = 2.5$$

$$x^2 - 14x + 49 + \frac{1}{9}x^2 + \frac{1}{3}kx - \frac{4}{3}x + \frac{1}{3}kx + k^2 - 4k - \frac{4}{3}x - 4k + 16 = 2.5$$

$$\frac{10}{9}x^2 + \left(-14 + \frac{1}{3}k - \frac{4}{3} + \frac{1}{3}k - \frac{4}{3}\right)x + k^2 - 4k - 4k + 49 + 16 - 2.5 = 0$$

$$\frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + k^2 - 8k + 62.5 = 0$$

Tangent $\Longrightarrow b^2 - 4ac = 0$ (tangent meets at one point hence one solution, so this is our hint to use the discriminant which talks about "how many" solutions)

$$\frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + (k^2 - 8k + 62.5) = 0$$

$$b^2 - 4ac = 0$$

$$\left[\frac{2}{3}k - \frac{50}{3}\right]^2 - 4\left(\frac{10}{9}\right)(k^2 - 8k + 62.5) = 0$$

$$\frac{4}{9}k^2 - \frac{200}{9}k + \frac{2500}{9} - \frac{40}{9}k^2 + \frac{320}{9}k - \frac{2500}{9} = 0$$

$$4k^2 - 200k + 2500 - 40k^2 + 320k - 2500 = 0$$

$$36k^2 - 120k = 0$$

$$4k(9k - 30) = 0$$

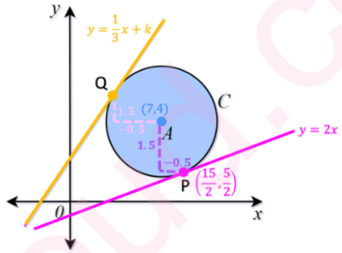
$$k = 0, k = \frac{30}{9} = \frac{10}{3}$$

$k = 0$ just picks up the $y = 2x$ tangent which we already know is the other tangent

$$k = \frac{10}{3}$$

## Way 2: Use symmetry

**(use this if you have one of the points on the circumference and want to do the question quickly. We should use this if the tangent equation has a fraction in it since it saves a lot of time, but be aware we can't do this method if we don't have another point on the circumference)**




We need point P first. We solve the radius equation PA which is $y = -3x + 25$ and tangent equation $y = 2x$ simultaneously to get this point P.

$$y = -3x + 25$$

$$y = \frac{1}{3}x$$

$$\frac{1}{3}x = -3x + 25$$

$$\frac{10}{3}x = 25$$

$$x = \frac{15}{2}$$

When $x = \frac{15}{2}$, $y = \frac{1}{3}\left(\frac{15}{2}\right) = \frac{5}{2}$

To get from P to A:

$x: -0.5$
$y: +1.5$

So, to get from A to Q we do the same thing:

$x: -0.5$
$y: +1.5$

$$\binom{7}{4} + \binom{-0.5}{1.5} = \binom{6.5}{5.5}$$

So $Q(6.5, 5.5)$

We have equation $y = \frac{1}{3}x + k$

Plug this point (6.5, 5.5) in

$$5.5 = \frac{1}{3}(6.5) + k$$

$$5.5 = \frac{13}{6} + k$$

$$k = \frac{10}{3}$$

**16.** The curve $C$ has equation $y = f(x)$ where

$$f(x) = ax^3 + 15x^2 - 39x + b$$

same as y

and $a$ and $b$ are constants.

Given

- the point (2, 10) lies on $C$ can plug this in
- the gradient of the curve at (2, 10) is –3 $\frac{dy}{dx} = -3$ when $x = 2$

same as $f'(x)$

(a) (i) show that the value of $a$ is –2

(ii) find the value of $b$.

**(4)**

(b) Hence show that $C$ has no stationary points.

**(3)**

(c) Write $f(x)$ in the form $(x - 4)Q(x)$ where $Q(x)$ is a quadratic expression to be found.

**(2)**

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = f(0.2x)$$

and the coordinate axes.

**(2)**

a) i) $f(x) = ax^3 + 15x^2 - 39x + b$

a and b are just constants, so we differentiate as normal

$f'(x) = 3ax^2 + 30x - 39$

$\frac{dy}{dx} = -3$ when $x = 2$

$-3 = 3a(2)^2 + 30(2) - 39$

$-3 = 12a + 60 - 39$

$12a = -24$

$a = -2$





P 6 6 5 8 5 A 0 4 2 4 4

**Question 16 continued**

ii)
So the equation becomes

$f(x) = -2x^3 + 15x^2 - 39x + b$

(2, 10) tells use that $x = 2, y = 10$

$10 = -2(2)^3 + 15(2)^2 - 39(2) + b$

$10 = -16 + 60 - 78 + b$

$b = 44$

b)
So the equation becomes

$f(x) = -2x^3 + 15x^2 - 39x + 44$

Stationary points when $f'(x) = 0$

$f'(x) = -6x^2 + 30x - 39$

$-6x^2 + 30x - 39 = 0$

$6x^2 - 30x + 39 = 0$

Use quadratic formula

$x = \dfrac{30 \pm \sqrt{(-30)^2 - 4(6)(39)}}{2(6)}$

$= \dfrac{30 \pm \sqrt{-36}}{-12}$ → discriminant is $< 0$ hence no solution

Question 16 continued

c) given $f(x) = (x-4)Q(x)$

$\underbrace{x-4}_{\text{factor}}$

Use polynomial division to find $Q(x)$
(or multiply out RHS & compare coefficients)

$$
\begin{array}{r}
-2x^2 + 7x - 11 \\
x-4 \overline{)\, -2x^3 + 15x^2 - 39x + 44} \\
\underline{-2x^3 + 8x^2} \quad\quad\quad\quad \\
7x^2 - 39x \quad\quad \\
\underline{7x^2 - 28x} \quad\quad \\
-11x \quad\quad
\end{array}
$$

$f(x) = (x-4)(-2x^2 + 7x - 11)$

d) When the curve intersects the y axis, $x = 0$:

$f(0) = -2(0)^3 + 15(0) - 39(0) + 44$

$f(0) = 44$ hence $(0, 44)$

When the curve intersects the $x$ axis $y = 0$:

$-2x^3 + 15x^2 - 39x + 44 = 0$

$(x-4)(-2x^2 + 7x - 11) = 0$

$x = 4$ ... no solution

hence $(4, 0)$

$f(0.2x)$ means $\div$ every $x$ value by $0.2$

$(0 \div 0.2, 44) \Rightarrow (0, 44)$

$(4 \div 0.2, 0) \Rightarrow (20, 0)$

**(Total for Question 16 is 11 marks)**

**TOTAL FOR PAPER IS 100 MARKS**